SAS/GLM AND SAS/MIXED PROCEDURES FOR THREE FORMULATIONS OF THE SAME MODEL

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Keywords: mixed model, centered variates, non-centered variated, polynomial regression, regression, row-column design.

Abstract:

Three formulations of the same response model are presented. One formulation is the standard textbook row-column model. The second model uses up to the sixth degree orthogonal polynomial for columns and up to the seventh degree orthogonal polynomial for rows. The third formulation uses the con-centered form of polynomials for rows and columns. The models relate to an eight row by a seven column designed experimental set of data. Identical results were obtained for a fixed effect analysis using the SAS/GLM procedure. Different results were obtained for the first two models using the SAS/MIXED procedure. Results for the non-centered polynomials were not obtained using the SAS/MIXED procedure. In an attempt to reconcile differences, two procedures of pooling row and column effects were tried and were partially successful. Some variations of the models are also included.

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ABSTRACT

Three formulations of the same response model are presented. One formulation is the standard textbook row-column model. The second model uses up to the sixth degree orthogonal polynomial for columns and up to the seventh degree orthogonal polynomial for rows. The third formulation uses the non-centered form of polynomials for rows and columns. The models relate to an eight row by a seven column designed experimental set of data. Identical results were obtained for a fixed effect analysis using the SAS/GLM procedure. Different results were obtained for the first two models using the SAS/Mixed procedure. Results for the non-centered polynomials were not obtained using the SAS/MIXED procedure. In an attempt to reconcile differences, two procedures of pooling row and column effects were tried and were partially successful. Some variations of the models are also included.

Key words: Mixed model, centered variates, non-centered variated, polynomial regression, row-column design

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INTRODUCTION

In an attempt to have a deeper understanding of the SAS/GLM and SAS/MIXED procedures, the response model for an experiment designed as a randomized complete block but laid out as an eight-row by seven-column experiment is formulated in three different ways. The models are identical except for the way they are formulated. The three models are:

Model 1:

$$Y_{hij} = \mu + \rho_h + \gamma_i + \tau_j + \varepsilon_{hij}$$

Model 2:

$$Y_{hij} = \mu + pc1 + pc2 + pc3 + pc3 + pc4 + pc5 + pc6 + pr1 + pr2 + pr3 + pr4 + pr5 + pr6 + pr7 + \tau_j + \varepsilon_{hij}$$

Model 3:

$$Y_{hij} = \mu + c1 + c2 +c3 + c4 + c5 + c6 + r1 + r2 + r3 + r4 + r5 + r6 + r7 + \tau_j + \varepsilon_{hij}$$

where μ is a general mean effect, ρ_h is the h^{th} row effect, γ_i is the i^{th} column effect, τ_j is the j^{th} treatment effect, ε_{hij} is a random error distributed with mean zero and variance σ_ε^2, pci is the i^{th} column non-centered polynomial regression coefficient, prh is the h^{th} non-centered row polynomial regression coefficient, ci is the i^{th} orthogonal polynomial regression coefficient, and rh is the h^{th} orthogonal polynomial regression coefficient. It may be observed that SAS/GLM analyses will be identical with regard the estimated error variance, solutions for treatment effects, and least squares treatment means for all three models. The solutions for these effects will not be the same when the SAS/MIXED procedure is used. An attempt to fix this dilemma is made by pooling all blocking sources of variation and to have only having blocking and error variance components. Some variations of the models are included by putting sigma restraints on the row and column effects.

THE SAS COMPUTER PROGRAM

The following SAS code was used to obtain the analyses (comments enclosed in /*-- --*/):

```
 /*The SAS code for obtaining standard textbook analysis and PRTA centered and
non-centered independent variates are: */
data colrow;
  infile 'colrow.dat';
  input Yield row col Trt;

/*--code for non-centered polynomials--*/
pc1= col ;pc2=col**2;pc3=col**3;pc4=col**4;pc5=col**5;pc6=col**6;

pr1= row ;pr2=row**2;pr3=row**3;pr4=row**4;pr5=row**5;
pr6=row**6; pr7 = row**7 ;proc print;
run; /*-- A data set called work.colrow is produced with the 6 + 7 = 13 non-
centered polynomial coefficients added.--*/

/*--code to construct orthogonal polynomials, centered values of independent
variates--*/
Proc iml;
/*--7 columns and up to 6th degree polynomials--*/
opn4=orpol(1:7,6);
opn4[,1]=(1:7)`;
 op4=opn4;       print op4; /*--prints orthogonal polynomial coefficients--*/
 create opn4 from opn4[colname={'col' 'c1' 'c2' 'c3' 'c4' 'c5' 'c6'}];
 append from opn4;
 close opn4;      run;
opn3=orpol(1:8,7);
 opn3[,1]=(1:8)`;
 op3=opn3;     print op3;
 create opn3 from opn3[colname={'row' 'r1' 'r2' 'r3' 'r4' 'r5'
 'r6' 'r7'}] ;
 append from opn3;
 close opn3;      run;
/*--merge in polynomial coefficients--*/
 data rcbig;
 set colrow;
 idx = _n_;
 proc sort data = rcbig;
  by col ;
 data rcbig ;
  merge rcbig opn4;
  by col ;
 proc sort data = rcbig;
  by row ;
 data rcbig ;
  merge rcbig opn3;
  by row ;
 proc sort data = rcbig ;
  by idx ;
run;
/*--rcbig is the original colrow data set with 13 columns of orthogonal
polynomial regression coefficients added--*/
```

SAS/GLM procedure for fixed effects

```
/*--Standard textbook ANOVA and means for row-column design--*/
Proc Glm data = rcbig;
 Class row col Trt ;
```

```
 Model Yield = row  col  Trt ;
 Lsmeans Trt;
run ;

/*--Orthogonal polynomial'regressions of rows and columns ANOVA and means--*/
 Proc Glm data = rcbig ;
  Class Trt row  col  ;
  Model Yield = c1 c2 c3 c4 c5 c6 r1 r2 r3 r4 r5 r6 r7 Trt ;
  lsmeans Trt ;
run;

/*-- Non-centered Polynomial regressions of rows and columns ANOVA and means--*/
Proc Glm data = work.colrow ;
  Class Trt row col;
  Model Yield = pc1 pc2 pc3 pc4 pc5 pc6 pr1 pr2 pr3 pr4 pr5 pr6 pr7 Trt;
  lsmeans Trt;
run;
```

SAS/MIXED procedure for random blocking effects

```
/*--Random blocking effects analysis for standard row-column procedure.--*/
Proc Mixed data = rcbig ;
 Class row col Trt ;
 Model Yield = Trt/solution ;
 Random row col;
 Lsmeans Trt ;
run;

/*--Random blocking effects using orthogonal polynomial coefficients--*/
Proc mixed data = rcbig ;
 Class Trt row col ;
 Model Yield = Trt/solution;
 Random c1 c2 c3 c4 c5 c6 r1 r2 r3 r4 r5 r6 r7 ;
 Lsmeans Trt;
run;

/*--Random blocking effects using non-centered regression coefficients--*/
Proc mixed data = work.colrow ;
 Class Trt row col ;
 Model Yield = Trt/solution;
 Random pc1 pc2 pc3 pc4 pc5 pc6 pr1 pr2 pr3 pr4 pr5 pr6 pr7 ;
 Lsmeans Trt;
run;
```

SAS/MIXED for pooling random blocking effects

```
\*--the command /type = toep(1) pools all effects in the RANDOM statement--*/
Proc mixed data = rcbig;
  Class Trt row col ;
  Model Yield = Trt/solution;
  Random row col/type = toep(1);
  Lsmeans Trt;
run;

Proc mixed data = rcbig;
 Class Trt row col;
```

```
 Model Yield=Trt/solution;
 Random  c1 c2 c3 c4 c5 c6 r1 r2 r3 r4 r5 r6 r7 /type = toep(1) ;
 Lsmeans Trt;
run;

 Proc mixed data = work.colrow;
  Class Trt row col;
  Model Yield = Trt/solution;
  Random pc1 pc2 pc3 pc4 pc5 pc6 pr1 pr2 pr3 pr4 pr5 pr6 pr7/type =
  toep(1);
  Lsmeans Trt;
run;

Proc mixed data = rcbig;
  Class Trt row col;
  Model Yield = Trt/solution;
  Random c1 c2 c3 c4 c5 c6/type = toep(1) s cl;
/*--the S and CL options give BLUP-like solutions for effects in the RANDOM
statement with a significance test for each effect; here only the column effects
are combined into one source of random blocking column effects--*/
  Random r1 r2 r3 r4 r5 r6 r7/type = toep(1) ;
  Lsmeans Trt ;
run ;

Proc mixed data = work.colrow ;
  Class Trt row col ;
  Model Yield = Trt/solution ;
  Random pc1 pc2 pc3 pc4 pc5 pc6/type = toep(1) s cl ;
  Random pr1 pr2 pr3 pr4 pr5 pr6 pr7/type = toep(1) ;
  Lsmeans Trt ; run ; quit;
```

ABBREVIATED OUTPUT FROM ABOVE PROGRAM FOR COMPUTING REGRESSION COEFFICIENTS AND CONSTRUCTING DATA SETS FOR ANALYSES

The following is the data set work.colrow with the non-centered regression coefficients appended to the data set:

OBS	YIELD	ROW	COL	TRT	PC1	PC2	PC3	PC4	PC5	PC6	PR1	PR2	PR3	PR4	PR5	PR6	PR7
1	1299.2	1	1	6	1	1	1	1	1	1	1	1	1	1	1	1	1
2	875.9	1	2	7	2	4	8	16	32	64	1	1	1	1	1	1	1
3	960.7	1	3	4	3	9	27	81	243	729	1	1	1	1	1	1	1
4	1004.0	1	4	3	4	16	64	256	1024	4096	1	1	1	1	1	1	1
5	1173.2	1	5	1	5	25	125	625	3125	15625	1	1	1	1	1	1	1
6	1031.9	1	6	2	6	36	216	1296	7776	46656	1	1	1	1	1	1	1
7	1421.1	1	7	5	7	49	343	2401	16807	117649	1	1	1	1	1	1	1
8	1369.2	2	1	2	1	1	1	1	1	1	2	4	8	16	32	64	128
9	844.2	2	2	5	2	4	8	16	32	64	2	4	8	16	32	64	128
10	968.7	2	3	6	3	9	27	81	243	729	2	4	8	16	32	64	128
11	975.5	2	4	7	4	16	64	256	1024	4096	2	4	8	16	32	64	128
12	1322.4	2	5	3	5	25	125	625	3125	15625	2	4	8	16	32	64	128
13	1172.6	2	6	1	6	36	216	1296	7776	46656	2	4	8	16	32	64	128

14	1418.9	2	7	4	7	49	343	2401	16807	117649	2	4	8	16	32	64	128
15	1169.5	3	1	1	1	1	1	1	1	1	3	9	27	81	243	729	2187
16	975.8	3	2	5	2	4	8	16	32	64	3	9	27	81	243	729	2187
17	873.4	3	3	3	3	9	27	81	243	729	3	9	27	81	243	729	2187
18	797.8	3	4	7	4	16	64	256	1024	4096	3	9	27	81	243	729	2187
19	1069.7	3	5	2	5	25	125	625	3125	15625	3	9	27	81	243	729	2187
20	1093.3	3	6	6	6	36	216	1296	7776	46656	3	9	27	81	243	729	2187
21	1169.6	3	7	4	7	49	343	2401	16807	117649	3	9	27	81	243	729	2187
22	1219.1	4	1	6	1	1	1	1	1	1	4	16	64	256	1024	4096	16384
23	971.7	4	2	1	2	4	8	16	32	64	4	16	64	256	1024	4096	16384
24	607.6	4	3	7	3	9	27	81	243	729	4	16	64	256	1024	4096	16384
25	1000.0	4	4	4	4	16	64	256	1024	4096	4	16	64	256	1024	4096	16384
26	1343.3	4	5	2	5	25	125	625	3125	15625	4	16	64	256	1024	4096	16384
27	999.4	4	6	5	6	36	216	1296	7776	46656	4	16	64	256	1024	4096	16384
28	1181.3	4	7	3	7	49	343	2401	16807	117649	4	16	64	256	1024	4096	16384
29	1120.0	5	1	6	1	1	1	1	1	1	5	25	125	625	3125	15625	78125
30	827.0	5	2	7	2	4	8	16	32	64	5	25	125	625	3125	15625	78125
31	671.9	5	3	4	3	9	27	81	243	729	5	25	125	625	3125	15625	78125
32	972.2	5	4	3	4	16	64	256	1024	4096	5	25	125	625	3125	15625	78125
33	1083.7	5	5	1	5	25	125	625	3125	15625	5	25	125	625	3125	15625	78125
34	1146.9	5	6	2	6	36	216	1296	7776	46656	5	25	125	625	3125	15625	78125
35	993.8	5	7	5	7	49	343	2401	16807	117649	5	25	125	625	3125	15625	78125
36	1031.5	6	1	7	1	1	1	1	1	1	6	36	216	1296	7776	46656	279936
37	846.5	6	2	2	2	4	8	16	32	64	6	36	216	1296	7776	46656	279936
38	667.8	6	3	4	3	9	27	81	243	729	6	36	216	1296	7776	46656	279936
39	853.6	6	4	3	4	16	64	256	1024	4096	6	36	216	1296	7776	46656	279936
40	1087.1	6	5	1	5	25	125	625	3125	15625	6	36	216	1296	7776	46656	279936
41	990.2	6	6	5	6	36	216	1296	7776	46656	6	36	216	1296	7776	46656	279936
42	1021.9	6	7	6	7	49	343	2401	16807	117649	6	36	216	1296	7776	46656	279936
43	1076.4	7	1	2	1	1	1	1	1	1	7	49	343	2401	16807	117649	823543
44	917.9	7	2	1	2	4	8	16	32	64	7	49	343	2401	16807	117649	823543
45	627.6	7	3	5	3	9	27	81	243	729	7	49	343	2401	16807	117649	823543
46	776.4	7	4	6	4	16	64	256	1024	4096	7	49	343	2401	16807	117649	823543
47	960.4	7	5	3	5	25	125	625	3125	15625	7	49	343	2401	16807	117649	823543
48	852.4	7	6	7	6	36	216	1296	7776	46656	7	49	343	2401	16807	117649	823543
49	1006.2	7	7	4	7	49	343	2401	16807	117649	7	49	343	2401	16807	117649	823543
50	1099.6	8	1	4	1	1	1	1	1	1	8	64	512	4096	32768	262144	2097152
51	947.4	8	2	5	2	4	8	16	32	64	8	64	512	4096	32768	262144	2097152
52	787.1	8	3	2	3	9	27	81	243	729	8	64	512	4096	32768	262144	2097152
53	898.3	8	4	1	4	16	64	256	1024	4096	8	64	512	4096	32768	262144	2097152
54	1174.9	8	5	3	5	25	125	625	3125	15625	8	64	512	4096	32768	262144	2097152
55	1003.3	8	6	6	6	36	216	1296	7776	46656	8	64	512	4096	32768	262144	2097152
56	947.6	8	7	7	7	49	343	2401	16807	117649	8	64	512	4096	32768	262144	2097152

The following are the centered orthogonal polynomial regression coefficients of order seven and eight respectively:

OP4

1	-0.566947	0.5455447	-0.408248	0.2417469	-0.109109	0.0328976
2	-0.377964	-9.68E-16	0.4082483	-0.564076	0.4364358	-0.197386
3	-0.188982	-0.327327	0.4082483	0.0805823	-0.545545	0.4934638
4	1.678E-16	-0.436436	-1.34E-15	0.4834938	5.182E-14	-0.657952
5	0.1889822	-0.327327	-0.408248	0.0805823	0.5455447	0.4934638
6	0.3779645	4.24E-16	-0.408248	-0.564076	-0.436436	-0.197386
7	0.5669467	0.5455447	0.4082483	0.2417469	0.1091089	0.0328976

OP3							
1	-0.540062	0.5400617	-0.43082	0.282038	-0.149786	0.0615457	-0.01707
2	-0.385758	0.0771517	0.3077287	-0.523785	0.4921546	-0.307729	0.119488
3	-0.231455	-0.231455	0.4308202	-0.120873	-0.363766	0.5539117	-0.358464
4	-0.077152	-0.385758	0.1846372	0.3626203	-0.32097	-0.307729	0.5974401
5	0.0771517	-0.385758	-0.184637	0.3626203	0.3209704	-0.307729	-0.59744
6	0.231455	-0.231455	-0.43082	-0.120873	0.3637664	0.5539117	0.3584641
7	0.3857584	0.0771517	-0.307729	-0.523785	-0.492155	-0.307729	-0.119488
8	0.5400617	0.5400617	0.4308202	0.282038	0.1497862	0.0615457	0.0170697

Output for General Linear Models Procedure, SAS/GLM

The standard row-column analysis is:

Dependent Variable: YIELD

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	19	1667110.584	87742.662	11.93	0.0001
Error	36	264666.041	7351.834		
Corrected Total	55	1931776.625			

R-Square	C.V.	Root MSE	YIELD Mean
0.862993	8.468638	85.74284	1012.475

Dependent Variable: YIELD

Source	DF	Type I SS	Mean Square	F Value	Pr > F
ROW	7	388314.902	55473.557	7.55	0.0001
COL	6	1159072.132	193178.689	26.28	0.0001
TRT	6	119723.549	19953.925	2.71	0.0281

Source	DF	Type III SS	Mean Square	F Value	Pr > F
ROW	7	388314.902	55473.557	7.55	0.0001
COL	6	1004920.232	167486.705	22.78	0.0001
TRT	6	119723.549	19953.925	2.71	0.0281

Least Squares Means

TRT	YIELD LSMEAN
1	1029.61023
2	1052.68030
3	1051.44268
4	1032.27664
5	997.48579
6	1023.02964
7	900.79972

The general linear models procedure, SAS/GLM, analysis using orthogonal polynomial regression coefficients is:

Dependent Variable: YIELD

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F

Model	19	1667110.584	87742.662	11.93	0.0001
Error	36	264666.041	7351.834		
Corrected Total	55	1931776.625			

R-Square	C.V.	Root MSE	YIELD Mean
0.862993	8.468638	85.74284	1012.475

Dependent Variable: YIELD

Source	DF	Type I SS	Mean Square	F Value	Pr > F
C1	1	92210.7457	92210.7457	12.54	0.0011
C2	1	454417.6117	454417.6117	61.81	0.0001
C3	1	395597.4533	395597.4533	53.81	0.0001
C4	1	31263.7325	31263.7325	4.25	0.0465
C5	1	170085.5372	170085.5372	23.14	0.0001
C6	1	15497.0521	15497.0521	2.11	0.1552
R1	1	278087.6327	278087.6327	37.83	0.0001
R2	1	21476.7478	21476.7478	2.92	0.0960
R3	1	43739.6582	43739.6582	5.95	0.0198
R4	1	1967.8963	1967.8963	0.27	0.6081
R5	1	20330.7758	20330.7758	2.77	0.1050
R6	1	11851.9481	11851.9481	1.61	0.2123
R7	1	10860.2434	10860.2434	1.48	0.2321
TRT	6	119723.5490	19953.9248	2.71	0.0281

Source	DF	Type III SS	Mean Square	F Value	Pr > F
C1	1	74565.8940	74565.8940	10.14	0.0030
C2	1	409999.6821	409999.6821	55.77	0.0001
C3	1	246578.5053	246578.5053	33.54	0.0001
C4	1	14325.9911	14325.9911	1.95	0.1713
C5	1	130068.7710	130068.7710	17.69	0.0002
C6	1	2178.7015	2178.7015	0.30	0.5895
R1	1	278087.6327	278087.6327	37.83	0.0001
R2	1	21476.7478	21476.7478	2.92	0.0960
R3	1	43739.6582	43739.6582	5.95	0.0198
R4	1	1967.8963	1967.8963	0.27	0.6081
R5	1	20330.7758	20330.7758	2.77	0.1050
R6	1	11851.9481	11851.9481	1.61	0.2123
R7	1	10860.2434	10860.2434	1.48	0.2321
TRT	6	119723.5490	19953.9248	2.71	0.0281

Least Squares Means

TRT	YIELD LSMEAN
1	1029.61023
2	1052.68030
3	1051.44268
4	1032.27664
5	997.48579
6	1023.02964
7	900.79972

The general linear models procedure using non-centered regression coefficients is:

Dependent Variable: YIELD

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	19	1667110.591	87742.663	11.93	0.0001
Error	36	264666.034	7351.834		
Corrected Total	55	1931776.625			

R-Square	C.V.	Root MSE	YIELD Mean
0.862993	8.468638	85.74284	1012.475

Dependent Variable: YIELD

Source	DF	Type I SS	Mean Square	F Value	Pr > F
PC1	1	92210.7457	92210.7457	12.54	0.0011
PC2	1	454417.6117	454417.6117	61.81	0.0001
PC3	1	395597.4533	395597.4533	53.81	0.0001
PC4	1	31263.7325	31263.7325	4.25	0.0465
PC5	1	170085.5372	170085.5372	23.14	0.0001
PC6	1	15497.0520	15497.0520	2.11	0.1552
PR1	1	278087.6327	278087.6327	37.83	0.0001
PR2	1	21476.7478	21476.7478	2.92	0.0960
PR3	1	43739.6582	43739.6582	5.95	0.0198
PR4	1	1967.8963	1967.8963	0.27	0.6081
PR5	1	20330.7758	20330.7758	2.77	0.1050
PR6	1	11851.9481	11851.9481	1.61	0.2123
PR7	1	10860.2508	10860.2508	1.48	0.2321
TRT	6	119723.5491	19953.9249	2.71	0.0281

Source	DF	Type III SS	Mean Square	F Value	Pr > F
PC1	1	161.3708	161.3708	0.02	0.8830
PC2	1	103.5728	103.5728	0.01	0.9062
PC3	1	1.9253	1.9253	0.00	0.9872
PC4	1	274.7329	274.7329	0.04	0.8478
PC5	1	1033.7543	1033.7543	0.14	0.7099
PC6	1	2178.7015	2178.7015	0.30	0.5895
PR1	1	17697.2553	17697.2553	2.41	0.1295
PR2	1	16193.4698	16193.4698	2.20	0.1465
PR3	1	14737.4810	14737.4810	2.00	0.1654
PR4	1	13480.9665	13480.9665	1.83	0.1841
PR5	1	12426.5481	12426.5481	1.69	0.2018
PR6	1	11559.7751	11559.7751	1.57	0.2179
PR7	1	10860.2508	10860.2508	1.48	0.2321
TRT	6	119723.5491	19953.9249	2.71	0.0281

The small F-values associated with the PRh regressions in the TYPE III but not TYPE I analysis is worthy of note but the explanation is not forth-coming. This may be another reason for not using non-centered independent variates.

Least Squares Means

TRT	YIELD LSMEAN

1	1029.61023
2	1052.68030
3	1051.44268
4	1032.27664
5	997.48579
6	1023.02964
7	900.79972

As can be seen from the above output for the three formulations of the same model, identical results are obtained. The error means squares, coefficients of variation, F-values for treatment means squares, least squares means, etc. are the same for all three formulations. This is as it should as all models are the same except for formulation.

SAS/MIXED, mixed model, output

The outputs from the above code for the three model formulations are given below. The output for the standard row-column analysis is:

Covariance Parameter Estimates (REML)

Cov Parm	Estimate
ROW	6871.4137762
COL	21140.553487
Residual	7346.5136281

Solution for Fixed Effects

Effect	TRT	Estimate	Std Error	DF	t	Pr > |t|
INTERCEPT		899.10048822	69.57963033	6	12.92	0.0001
TRT	1	132.14004949	45.34032988	36	2.91	0.0061
TRT	2	155.33549950	45.08932245	36	3.45	0.0015
TRT	3	151.96421589	46.13745248	36	3.29	0.0022
TRT	4	131.23727827	45.57741737	36	2.88	0.0067
TRT	5	97.15793887	44.35775598	36	2.19	0.0351
TRT	6	125.78660047	44.72186499	36	2.81	0.0079
TRT	7	0.00000000	.	.	.	.

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
TRT	6	36	2.82	0.0238

Least Squares Means

Effect	TRT	LSMEAN	Std Error	DF	t	Pr > |t|
TRT	1	1031.2405377	70.20281239	36	14.69	0.0001
TRT	2	1054.4359877	69.86373179	36	15.09	0.0001
TRT	3	1051.0647041	70.70420090	36	14.87	0.0001
TRT	4	1030.3377665	70.70837157	36	14.57	0.0001
TRT	5	996.25842709	70.37937739	36	14.16	0.0001
TRT	6	1024.8870887	70.09374499	36	14.62	0.0001
TRT	7	899.10048822	69.57963033	36	12.92	0.0001

The output for the program when using orthogonal polynomial regression coefficients is:

Covariance Parameter Estimates (REML)

Cov Parm	Estimate
C1	8699.0648303
C2	60982.708343
C3	38400.202506
C4	1128.2130768
C5	18946.293934
C6	0.00000000
R1	38716.745270
R2	2057.9861515
R3	5238.4120453
R4	0.00000000
R5	1894.2761066
R6	683.02312905
R7	541.35286498
Residual	7070.5569655

Solution for Fixed Effects

Effect	TRT	Estimate	Std Error	DF	t	Pr > |t|
INTERCEPT		896.05281368	29.98769912	36	29.88	0.0001
TRT	1	133.81011175	44.04220522	36	3.04	0.0044
TRT	2	156.03632988	43.32061498	36	3.60	0.0009
TRT	3	163.78086694	44.46401752	36	3.68	0.0008
TRT	4	141.62832821	43.51295769	36	3.25	0.0025
TRT	5	93.51438290	42.99605314	36	2.17	0.0363
TRT	6	126.18528459	43.72895576	36	2.89	0.0066
TRT	7	0.00000000	.	.	.	.

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
TRT	6	36	3.37	0.0097

Least Squares Means

Effect	TRT	LSMEAN	Std Error	DF	t	Pr > |t|
TRT	1	1029.8629254	31.70833227	36	32.48	0.0001
TRT	2	1052.0891436	30.72544216	36	34.24	0.0001
TRT	3	1059.8336806	32.31332979	36	32.80	0.0001
TRT	4	1037.6811419	32.30106702	36	32.13	0.0001
TRT	5	989.56719658	31.47964823	36	31.44	0.0001
TRT	6	1022.2380983	31.51654931	36	32.43	0.0001
TRT	7	896.05281368	29.98769912	36	29.88	0.0001

Note that the analysis using non-centered regression coefficients could not be completed as the process of iteration stopped because of too many likelihood evaluations. Solutions for variance components obtained when the program stopped are given below:

REML Estimation Iteration History

Iteration	Evaluations	Objective	Criterion
1	4	553.76970803	5.7709199E12
2	3	553.12602516	.
3	1	553.07970755	.
4	44	553.07970755	2.8822242E18
5	45	553.07970755	2.8822242E18
6	42	553.07970755	2.8822242E18

Stopped because of too many likelihood evaluations.

Covariance Parameters at
Last REML Iteration

Cov Parm	Estimate
PC1	109707.66765
PC2	0.00000000
PC3	530.22870298
PC4	0.00000004
PC5	0.19439022
PC6	0.00001724
PR1	-0.00000000
PR2	358.30397214
PR3	-0.00000000
PR4	0.13455887
PR5	0.00000000
PR6	0.00000041
PR7	0.00000000
Residual	11136.667517

The mixed model analysis using REML solutions for the variance components for models 1 and 2 are not the same as they were for the fixed effects analysis. The values for the residual variance component, solutions for treatment effects and means, and standard errors are not the same for models 1 and 2. No results for the statistics could be obtained for the third model using non-centered polynomial regression coefficients.

Pooling blocking sources of variation

Since the SAS/MIXED procedure using REML solutions for variance components may have some instability when single or small numbers of degrees of freedom are associated with an effect, it is of interest to determine what happens when the sources of variation are pooled. Outputs are obtained when all row and column effects are pooled into one source of variation and when all column effects are pooled and when all row effects are pooled.

For the standard row-column analysis the row and column effects are pooled into one source of variation. The results are:

The MIXED Procedure

Covariance Parameter Estimates (REML)

Cov Parm	Estimate
Variance	13492.480682
Residual	7340.1253146

Model Fitting Information for YIELD

Description	Value
Null Model LRT Chi-Square	40.8107
Null Model LRT DF	1.0000
Null Model LRT P-Value	0.0000

Solution for Fixed Effects

Effect	TRT	Estimate	Std Error	DF	t	Pr > |t|
INTERCEPT		898.20891657	67.63430122	6	13.28	0.0001
TRT	1	133.87514177	45.24637775	36	2.96	0.0054
TRT	2	157.13595676	45.00962590	36	3.49	0.0013
TRT	3	152.65510706	46.02072221	36	3.32	0.0021
TRT	4	131.12978052	45.48108923	36	2.88	0.0066
TRT	5	97.41261800	44.29774009	36	2.20	0.0344
TRT	6	127.65397989	44.65253987	36	2.86	0.0070
TRT	7	0.00000000	.	.	.	.

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
TRT	6	36	2.87	0.0216

Least Squares Means

Effect	TRT	LSMEAN	Std Error	DF	t	Pr > |t|
TRT	1	1032.0840583	68.25251339	36	15.12	0.0001
TRT	2	1055.3448733	67.91693010	36	15.54	0.0001
TRT	3	1050.8640236	68.75267205	36	15.28	0.0001
TRT	4	1029.3386971	68.75659577	36	14.97	0.0001
TRT	5	995.62153457	68.43092713	36	14.55	0.0001
TRT	6	1025.8628965	68.14700804	36	15.05	0.0001
TRT	7	898.20891657	67.63430122	36	13.28	0.0001

All row and column orthogonal polynomial regression coefficient effects have been pooled into one source of variation for the following output:

Covariance Parameter Estimates (REML)

Cov Parm	Estimate
Variance	13492.480682
Residual	7340.1253146

Model Fitting Information for YIELD

Description	Value
Null Model LRT Chi-Square	40.8107
Null Model LRT DF	1.0000
Null Model LRT P-Value	0.0000

Solution for Fixed Effects

Effect	TRT	Estimate	Std Error	DF	t	Pr > |t|
INTERCEPT		898.20891657	30.98937520	36	28.98	0.0001
TRT	1	133.87514177	45.24637775	36	2.96	0.0054
TRT	2	157.13595676	45.00962590	36	3.49	0.0013
TRT	3	152.65510706	46.02072221	36	3.32	0.0021
TRT	4	131.12978052	45.48108923	36	2.88	0.0066
TRT	5	97.41261800	44.29774009	36	2.20	0.0344
TRT	6	127.65397989	44.65253987	36	2.86	0.0070
TRT	7	0.00000000	.	.	.	.

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
TRT	6	36	2.87	0.0216

Least Squares Means

Effect	TRT	LSMEAN	Std Error	DF	t	Pr > |t|
TRT	1	1032.0840583	32.31637755	36	31.94	0.0001
TRT	2	1055.3448733	31.60145675	36	33.40	0.0001
TRT	3	1050.8640236	33.35974503	36	31.50	0.0001
TRT	4	1029.3386971	33.36783086	36	30.85	0.0001
TRT	5	995.62153457	32.69150443	36	30.46	0.0001
TRT	6	1025.8628965	32.09294906	36	31.97	0.0001
TRT	7	898.20891657	30.98937520	36	28.98	0.0001

The following is the output from the above program using non-centered polynomial regression coefficients. Note that the process did not converge and hence the analysis was not completed.

REML Estimation Iteration History

Iteration	Evaluations	Objective	Criterion
0	1	574.58898357	
1	4	574.58898357	.
2	44	574.58898357	1.6685075E24
3	44	574.58898357	1.6685075E24

Did not converge.

Covariance Parameters at
Last REML Iteration

Cov Parm	Estimate
Variance	-0.00000000
Residual	33834.717857

Instead of pooling all row and column effects into a single source, the following outputs were obtained by pooling all column effects and then by pooling all row effects. These outputs are to be compared to the one obtained for the standard row-column mixed model analysis. The following was obtained using orthogonal polynomial regression coefficients:

Covariance Parameter Estimates (REML)

Cov Parm	Estimate
Variance	21140.553487
Variance	6871.4137762
Residual	7346.5136281

Solution for Fixed Effects

Effect	TRT	Estimate	Std Error	DF	t	Pr > |t|
INTERCEPT		899.10048822	31.02126956	36	28.98	0.0001
TRT	1	132.14004949	45.34032988	36	2.91	0.0061
TRT	2	155.33549950	45.08932245	36	3.45	0.0015
TRT	3	151.96421589	46.13745248	36	3.29	0.0022
TRT	4	131.23727827	45.57741737	36	2.88	0.0067
TRT	5	97.15793887	44.35775598	36	2.19	0.0351
TRT	6	125.78660047	44.72186499	36	2.81	0.0079
TRT	7	0.00000000	.	.	.	.

Solution for Random Effects

Effec	Estimate	SE Pred	DF	t	Pr > |t|	Alpha	Lower
C1	95.25373904	30.35714715	36	3.14	0.0034	0.05	33.6866
C2	235.66610875	32.08221739	36	7.35	0.0001	0.05	170.6004
C3	-188.5512026	33.34483673	36	-5.65	0.0001	0.05	-256.178
C4	46.29688550	33.39010108	36	1.39	0.1741	0.05	-21.4214
C5	134.52422440	32.55846675	36	4.13	0.0002	0.05	68.4926
C6	16.08554898	30.63580700	36	0.53	0.6028	0.05	-46.0467
R1	-172.9069803	30.17358957	36	-5.73	0.0001	0.05	-234.102
R2	48.05138909	30.17358957	36	1.59	0.1200	0.05	-13.1435
R3	68.57398392	30.17358957	36	2.27	0.0291	0.05	7.3791
R4	14.54530382	30.17358957	36	0.48	0.6327	0.05	-46.6496
R5	46.75183527	30.17358957	36	1.55	0.1300	0.05	-14.4430
R6	-35.69575387	30.17358957	36	-1.18	0.2446	0.05	-96.8906
R7	34.16972374	30.17358957	36	1.13	0.2649	0.05	-27.0252

Solution for Random Effects

Upper

```
156.8209
300.7319
-120.925
114.0151
200.5559
 78.2178
-111.712
109.2463
129.7689
 75.7402
107.9467
 25.4991
 95.3646
```

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
TRT	6	36	2.82	0.0238

Least Squares Means

Effect	TRT	LSMEAN	Std Error	DF	t	Pr > |t|
TRT	1	1031.2405377	32.39489274	36	31.83	0.0001
TRT	2	1054.4359877	31.65336045	36	33.31	0.0001
TRT	3	1051.0647041	33.46756987	36	31.41	0.0001
TRT	4	1030.3377665	33.47638001	36	30.78	0.0001
TRT	5	996.25842709	32.77576804	36	30.40	0.0001
TRT	6	1024.8870887	32.15784966	36	31.87	0.0001
TRT	7	899.10048822	31.02126956	36	28.98	0.0001

When the PCi are pooled and the PRh are pooled, there were too many likelihood evaluations to allow the analysis to be completed. The following output is what was obtained:

REML Estimation Iteration History

Iteration	Evaluations	Objective	Criterion
0	1	574.58898357	
1	4	572.63477738	.
2	44	572.63477738	3.2402166E24
3	44	572.63477738	3.2402166E24
4	44	572.63477738	3.2402166E24

Stopped because of too many likelihood evaluations.

Covariance Parameters at
Last REML Iteration

Cov Parm	Estimate
Variance	0.00070947
Variance	0.00000000
Residual	27661.866326

The least squares means are the same for models 1 and 2 under both pooling procedures. When row regressions are pooled and column regressions are pooled, the least squares means are identical to the standard row-column mixed effects case. However, the standard errors of lsmeans for model 2 are less than half of those listed for model 1. Models 1 and 2 give the same estimates of treatment effects and the same standard errors. The estimate of the intercept is the same but the standard errors differ for the two models. Thus, the pooling procedure was only partiaslly successful for obtaining the same mixed model results for the two of the three formulations.

SOME MODEL VARIATIONS

To further investigate the effects of different model formulations, consider the following three procedures for placing a sigma restraint on the row and column effects. The first variation is to subtract the last row effect from each of the others and the same for column effects. This is the default for SAS/GLM procedures. Denote these differences of effects xci for columns and as xrj for rows. The second variation of the model formulation is to use the Helmert contrast matrices for row and column effects. These are denoted as cci for column contrasts and as crj for row contrasts in the following code. The third variation is to use orthogonal polynomial coefficients for column contrasts, pci, and for row contrasts, prj.

SAS code for preceding variations

The SAS code for the above three formulations is:

```
proc iml;
  opn7 =orpol(1:7,6);
  opn7[,1] = (1:7)`;
  op7 =opn7;
  create opn7 from opn7[colname={'COL' 'C1' 'C2' 'C3' 'C4' 'C5'
'C6' }]; append from opn7;
  close opn7; run;
  opn8 = orpol(1:8,7) ;
  opn8[,1]=(1:8)`;
  op8 = opn8;
  create opn8 from opn8[colname={'ROW' 'R1' 'R2' 'R3' 'R4' 'R5'
'R6' 'R7'}];  append from opn8;
close opn8;  run;
data colrow1;
  infile 'colrow.dat';
  input yield row col treat;
data crbig ; set colrow1;
  idx = _n_;
run;
proc sort data =  crbig;
```

```
  by col; run;
data  crbig;
  merge  crbig opn7;
  by col;      run;
proc sort data =  crbig;
  by row;  run;
data  crbig;
  merge  crbig opn8;
  by row;  run;
proc sort data =  crbig;
  by idx;  run;

data colrow1; set colrow1;
xr1=((row=1)-(row=8))/sqrt(2); xr2=((row=2)-(row=8))/sqrt(2);
xr3=((row=3)-(row=8))/sqrt(2); xr4=((row=4)-(row=8))/sqrt(2);
xr5=((row=5)-(row=8))/sqrt(2); xr6=((row=6)-(row=8))/sqrt(2);
xr7=((row=7)-(row=8))/sqrt(2);
xc1=((col=1)-(col=7))/sqrt(2); xc2=((col=2)-(col=7))/sqrt(2);
xc3=((col=3)-(col=7))/sqrt(2); xc4=((col=4)-(col=7))/sqrt(2);
xc5=((col=5)-(col=7))/sqrt(2); xc6=((col=6)-(col=7))/sqrt(2);
cr1=(1*(row=1)-1*(row=2)+0*(row=3)+0*(row=4)+0*(row=5)+0*(row=6)
      +0*(row=7)+0*(row=8))/sqrt(2);
cr2=(1*(row=1)+1*(row=2)-2*(row=3)+0*(row=4)+0*(row=5)+0*(row=6)
     +0*(row=7)+0*(row=8))/sqrt(6);
cr3=(1*(row=1)+1*(row=2)+1*(row=3)-3*(row=4)+0*(row=5)+0*(row=6)
     +0*(row=7)+0*(row=8))/sqrt(12);
cr4=(1*(row=1)+1*(row=2)+1*(row=3)+1*(row=4)-4*(row=5)+0*(row=6)
     +0*(row=7)+0*(row=8))/sqrt(20);
cr5=(1+(row=1)+1*(row=2)+1*(row=3)+1*(row=4)+1*(row=5)-5*(row=6)
     +0*(row=7)+0*(row=8))/sqrt(30);
cr6=(1*(row=1)+1*(row=2)+1*(row=3)+1*(row=4)+1*(row=5)+1*(row=6)
     -6*(row=7)+0*(row=8))/sqrt(42);
cr7=(1*(row=1)+1*(row=2)+1*(row=3)+1*(row=4)+1*(row=5)+1*(row=6)
     +1*(row=7)-7*(row=8))/sqrt(56);
cc1=(1*(col=1)-1*(col=2)+0*(col=3)+0*(col=4)+0*(col=5)+0*(col=6)
     +0*(col=7))/sqrt(2);
cc2=(1*(col=1)+1*(col=2)-2*(col=3)+0*(col=4)+0*(col=5)+0*(col=6)
     +0*(col=7))/sqrt(6);
cc3=(1*(col=1)+1*(col=2)+1*(col=3)-3*(col=4)+0*(col=5)+0*(col=6)
     +0*(col=7))/sqrt(12);
cc4=(1*(col=1)+1*(col=2)+1*(col=3)+1*(col=4)-4*(col=5)+0*(col=6)
     +0*(col=7))/sqrt(20);
cc5=(1*(col=1)+1*(col=2)+1*(col=3)+1*(col=4)+1*(col=5)-5*(col=6)
     +0*(col=7))/sqrt(30);
cc6=(1*(col=1)+1*(col=2)+1*(col=3)+1*(col=4)+1*(col=5)+1*(col=6)
     -6*(col=7))/sqrt(42);
pc1=(-3*(col=1)-2*(col=2)-1*(col=3)+0*(col=4)+1*(col=5)+2*(col=6)
+3*(col=7))/sqrt(28);
pc2=(5*(col=1)+0*(col=2)-3*(col=3)-4*(col=4)-3*(col=5)+0*(col=6)
+5*(col=7))/sqrt(84);
pc3=(-1*(col=1)+1*(col=2)+1*(col=3)+0*(col=4)-1*(col=5)-1*(col=6)
+1*(col=7))/sqrt(6);
pc4=(3*(col=1)-7*(col=2)+1*(col=3)+6*(col=4)+1*(col=5)-7*(col=6)
+3*(col=7))/sqrt(154);
pc5=(-1*(col=1)+4*(col=2)-5*(col=3)+0*(col=4)+5*(col=5)-4*(col=6)
+1*(col=7))/sqrt(84);
pc6=(1*(col=1)-6*(col=2)+15*(col=3)-20*(col=4)+15*(col=5)-6*(col=6)
```

```
+1*(col=7))/sqrt(924);
pr1=(-7*(row=1)-5*(row=2)-3*(row=3)-1*(row=4)+1*(row=5)+3*(row=6)
+5*(row=7)+7*(row=8))/sqrt(168);
pr2=(7*(row=1)-1*(row=2)-3*(row=3)-5*(row=4)-5*(row=5)-3*(row=6)
+1*(row=7)+7*(row=8))/sqrt(168);
pr3=(-7*(row=1)+5*(row=2)+7*(row=3)+3*(row=4)-3*(row=5)-7*(row=6)
-5*(row=7)+7*(row=8))/sqrt(264);
pr4=(7*(row=1)-13*(row=2)-3*(row=3)+9*(row=4)+9*(row=5)-3*(row=6)
-13*(row=7)+7*(row=8))/sqrt(616);
pr5=(-7*(row=1)+23*(row=2)-15*(row=3)-17*(row=4)+15*(row=5)+17*(row=6)
-23*(row=7)+7*(row=8))/sqrt(2184);
pr6=(1*(row=1)-5*(row=2)+9*(row=3)-5*(row=4)-5*(row=5)+9*(row=6)
-5*(row=7)+1*(row=8))/sqrt(264);
pr7=(-1*(row=1)+7*(row=2)-21*(row=3)+35*(row=4)-35*(row=5)+21*(row=6)
-7*(row=7)+1*(row=8))/sqrt(3432);

proc glm data = colrow1; class row col treat;
   model yield= cc1 cc2 cc3 cc4 cc5 cc6 cr1 cr2 cr3 cr4 cr5 cr6 cr6
   cr7 treat; lsmeans treat; run;
proc glm data = colrow1; class row col treat;
  model yield = xc1 xc2 xc3 xc4 xc5 xc6 xr1 xr2 xr3 xr4 xr5 xr6
  xr7 treat; lsmeans treat; run;
proc glm data = colrow1; class row col treat;
  model yield = pc1 pc2 pc3 pc4 pc5 pc6 pr1 pr2 pr3 pr4 pr5 pr6
  pr7 treat; lsmeans treat; run;
proc mixed data = crbig;
   class row col treat;
   model yield = treat;
   random C1 C2 C3 C4 C5 C6/TYPE = TOEP(1);
   random R1 R2 R3 R4 R5 R6 R7/TYPE = TOEP(1);
   lsmeans treat;  run;
proc mixed data = colrow1;
   class row col treat;
   model yield = treat;
   random cr1 cr2 cr3 cr4 cr5 cr6 cr7/type = toep(1);
   random cc1 cc2 cc3 cc4 cc5 cc6/type = toep(1);
   lsmeans treat; run;
proc mixed data = colrow1;
   class row col treat;
   model yield = treat;
   random xr1 xr2 xr3 xr4 xr5 xr6 xr7/type = toep(1);
   random xc1 xc2 xc3 xc4 xc5 xc6/type = toep(1);
   lsmeans treat; run;
 proc mixed data = colrow1; class row col treat;
   model yield = treat;
   random pc1 pc2 pc3 pc4 pc5 pc6/type = toep(1);
   random pr1 pr2 pr3 pr4 pr5 pr6 pr7/type = toep(1);
   lsmeans treat; run; quit;
```

Output for the above code

The output for the above program in abbreviated form is given below:

```
                       General Linear Models Procedure
                           Class Level Information
                      Class    Levels    Values
```

ROW	8	1 2 3 4 5 6 7 8
COL	7	1 2 3 4 5 6 7
TREAT	7	1 2 3 4 5 6 7

Dependent Variable: YIELD

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	19	1667110.584	87742.662	11.93	0.0001
Error	36	264666.041	7351.834		
Corrected Total	55	1931776.625			

R-Square	C.V.	Root MSE	YIELD Mean
0.862993	8.468638	85.74284	1012.475

Dependent Variable: YIELD

Source	DF	Type I SS	Mean Square	F Value	Pr > F
CC1	1	296507.4756	296507.4756	40.33	0.0001
CC2	1	378305.7852	378305.7852	51.46	0.0001
CC3	1	8860.8051	8860.8051	1.21	0.2796
CC4	1	291154.5006	291154.5006	39.60	0.0001
CC5	1	20199.6802	20199.6802	2.75	0.1061
CC6	1	164043.8858	164043.8858	22.31	0.0001
CR1	1	6666.4464	6666.4464	0.91	0.3473
CR2	1	56415.3450	56415.3450	7.67	0.0088
CR3	1	12371.1471	12371.1471	1.68	0.2028
CR4	1	66315.7786	66315.7786	9.02	0.0048
CR5	1	102146.6298	102146.6298	13.89	0.0007
CR6	1	135828.4098	135828.4098	18.48	0.0001
CR7	1	8571.1454	8571.1454	1.17	0.2874
TREAT	6	119723.5490	19953.9248	2.71	0.0281

Source	DF	Type III SS	Mean Square	F Value	Pr > F
CC1	1	195738.8805	195738.8805	26.62	0.0001
CC2	1	362303.8626	362303.8626	49.28	0.0001
CC3	1	6871.8714	6871.8714	0.93	0.3401
CC4	1	156435.5033	156435.5033	21.28	0.0001
CC5	1	21198.8766	21198.8766	2.88	0.0981
CC6	1	148642.8207	148642.8207	20.22	0.0001
CR1	1	6666.4464	6666.4464	0.91	0.3473
CR2	1	56415.3450	56415.3450	7.67	0.0088
CR3	1	12371.1471	12371.1471	1.68	0.2028
CR4	1	66315.7786	66315.7786	9.02	0.0048
CR5	1	102146.6298	102146.6298	13.89	0.0007
CR6	1	135828.4098	135828.4098	18.48	0.0001
CR7	1	8571.1454	8571.1454	1.17	0.2874
TREAT	6	119723.5490	19953.9248	2.71	0.0281

Least Squares Means

TREAT	YIELD LSMEAN
1	1029.61023
2	1052.68030

3	1051.44268
4	1032.27664
5	997.48579
6	1023.02964
7	900.79972

Dependent Variable: YIELD

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	19	1667110.584	87742.662	11.93	0.0001
Error	36	264666.041	7351.834		
Corrected Total	55	1931776.625			

R-Square	C.V.	Root MSE	YIELD Mean
0.862993	8.468638	85.74284	1012.475

Dependent Variable: YIELD

Source	DF	Type I SS	Mean Square	F Value	Pr > F
XC1	1	3138.8006	3138.8006	0.43	0.5176
XC2	1	355713.5502	355713.5502	48.38	0.0001
XC3	1	548568.7251	548568.7251	74.62	0.0001
XC4	1	49171.6501	49171.6501	6.69	0.0139
XC5	1	197203.7340	197203.7340	26.82	0.0001
XC6	1	5275.6725	5275.6725	0.72	0.4025
XR1	1	58864.3457	58864.3457	8.01	0.0076
XR2	1	54922.7010	54922.7010	7.47	0.0097
XR3	1	18553.6019	18553.6019	2.52	0.1209
XR4	1	2201.7646	2201.7646	0.30	0.5876
XR5	1	45458.3147	45458.3147	6.18	0.0177
XR6	1	84731.5619	84731.5619	11.53	0.0017
XR7	1	123582.6123	123582.6123	16.81	0.0002
TREAT	6	119723.5490	19953.9248	2.71	0.0281

Source	DF	Type III SS	Mean Square	F Value	Pr > F
XC1	1	196834.2893	196834.2893	26.77	0.0001
XC2	1	59226.1660	59226.1660	8.06	0.0074
XC3	1	498885.8260	498885.8260	67.86	0.0001
XC4	1	73329.4082	73329.4082	9.97	0.0032
XC5	1	82119.1151	82119.1151	11.17	0.0019
XC6	1	5942.9005	5942.9005	0.81	0.3746
XR1	1	75199.9601	75199.9601	10.23	0.0029
XR2	1	158138.8458	158138.8458	21.51	0.0001
XR3	1	623.0450	623.0450	0.08	0.7726
XR4	1	9022.0826	9022.0826	1.23	0.2753
XR5	1	12063.4826	12063.4826	1.64	0.2084
XR6	1	56587.2858	56587.2858	7.70	0.0087
XR7	1	123582.6123	123582.6123	16.81	0.0002
TREAT	6	119723.5490	19953.9248	2.71	0.0281

Least Squares Means

TREAT	YIELD LSMEAN
1	1029.61023

2	1052.68030
3	1051.44268
4	1032.27664
5	997.48579
6	1023.02964
7	900.79972

General Linear Models Procedure

Dependent Variable: YIELD

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	19	1667110.584	87742.662	11.93	0.0001
Error	36	264666.041	7351.834		
Corrected Total	55	1931776.625			

R-Square	C.V.	Root MSE	YIELD Mean
0.862993	8.468638	85.74284	1012.475

General Linear Models Procedure

Dependent Variable: YIELD

Source	DF	Type I SS	Mean Square	F Value	Pr > F
PC1	1	92210.7457	92210.7457	12.54	0.0011
PC2	1	454417.6117	454417.6117	61.81	0.0001
PC3	1	395597.4533	395597.4533	53.81	0.0001
PC4	1	31263.7325	31263.7325	4.25	0.0465
PC5	1	170085.5372	170085.5372	23.14	0.0001
PC6	1	15497.0521	15497.0521	2.11	0.1552
PR1	1	278087.6327	278087.6327	37.83	0.0001
PR2	1	14625.3625	14625.3625	1.99	0.1670
PR3	1	46287.7200	46287.7200	6.30	0.0167
PR4	1	1144.7659	1144.7659	0.16	0.6955
PR5	1	20732.3931	20732.3931	2.82	0.1018
PR6	1	14643.3660	14643.3660	1.99	0.1667
PR7	1	12793.6620	12793.6620	1.74	0.1954
TREAT	6	119723.5490	19953.9248	2.71	0.0281

Source	DF	Type III SS	Mean Square	F Value	Pr > F
PC1	1	74565.8940	74565.8940	10.14	0.0030
PC2	1	409999.6821	409999.6821	55.77	0.0001
PC3	1	246578.5053	246578.5053	33.54	0.0001
PC4	1	14325.9911	14325.9911	1.95	0.1713
PC5	1	130068.7710	130068.7710	17.69	0.0002
PC6	1	2178.7015	2178.7015	0.30	0.5895
PR1	1	285290.2474	285290.2474	38.81	0.0001
PR2	1	21200.2280	21200.2280	2.88	0.0981
PR3	1	45907.1322	45907.1322	6.24	0.0172
PR4	1	1261.8663	1261.8663	0.17	0.6811
PR5	1	23530.5978	23530.5978	3.20	0.0820
PR6	1	14714.4167	14714.4167	2.00	0.1657
PR7	1	12793.6620	12793.6620	1.74	0.1954
TREAT	6	119723.5490	19953.9248	2.71	0.0281

Least Squares Means

TREAT YIELD

	LSMEAN
1	1029.61023
2	1052.68030
3	1051.44268
4	1032.27664
5	997.48579
6	1023.02964
7	900.79972

Covariance Parameter Estimates (REML)

Cov Parm	Estimate
Variance	21140.553487
Variance	6871.4137762
Residual	7346.5136281

Least Squares Means

Effect	TREAT	LSMEAN	Std Error	DF	t	Pr > |t|
TREAT	1	1031.2405377	32.39489274	36	31.83	0.0001
TREAT	2	1054.4359877	31.65336045	36	33.31	0.0001
TREAT	3	1051.0647041	33.46756987	36	31.41	0.0001
TREAT	4	1030.3377665	33.47638001	36	30.78	0.0001
TREAT	5	996.25842709	32.77576804	36	30.40	0.0001
TREAT	6	1024.8870887	32.15784966	36	31.87	0.0001
TREAT	7	899.10048822	31.02126956	36	28.98	0.0001

Covariance Parameter Estimates (REML)

Cov Parm	Estimate
Variance	6871.4137762
Variance	21140.553487
Residual	7346.5136281

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
TREAT	6	36	2.82	0.0238

Least Squares Means

Effect	TREAT	LSMEAN	Std Error	DF	t	Pr > |t|
TREAT	1	1012.1079778	32.85996438	36	30.80	0.0001
TREAT	2	1035.3034278	32.12916761	36	32.22	0.0001
TREAT	3	1031.9321442	33.91793650	36	30.42	0.0001
TREAT	4	1011.2052066	33.92662969	36	29.81	0.0001
TREAT	5	977.12586715	33.23551044	36	29.40	0.0001
TREAT	6	1005.7545288	32.62630041	36	30.83	0.0001
TREAT	7	879.96792829	31.50662388	36	27.93	0.0001

Covariance Parameter Estimates (REML)

Cov Parm	Estimate

Variance	13145.590992
Variance	35461.750465
Residual	7387.1090815

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
TREAT	6	36	2.83	0.0231

Least Squares Means

Effect	TREAT	LSMEAN	Std Error	DF	t	Pr > |t|
TREAT	1	1032.9689192	32.49176889	36	31.79	0.0001
TREAT	2	1056.0304063	31.75183976	36	33.26	0.0001
TREAT	3	1051.2610567	33.52740601	36	31.36	0.0001
TREAT	4	1027.8192838	33.60306533	36	30.59	0.0001
TREAT	5	995.11038223	32.85365858	36	30.29	0.0001
TREAT	6	1025.3202068	32.22839047	36	31.81	0.0001
TREAT	7	898.81474495	31.10027285	36	28.90	0.0001

Cov Parm	Estimate
Variance	21138.394791
Variance	7140.1502090
Residual	7354.5439197

Tests of Fixed Effects

Source	NDF	DDF	Type III F	Pr > F
TREAT	6	36	2.81	0.0239

Least Squares Means

Effect	TREAT	LSMEAN	Std Error	DF	t	Pr > |t|
TREAT	1	1032.1278183	32.41780210	36	31.84	0.0001
TREAT	2	1055.3234105	31.67604755	36	33.32	0.0001
TREAT	3	1051.9497115	33.49083120	36	31.41	0.0001
TREAT	4	1031.2210261	33.49964317	36	30.78	0.0001
TREAT	5	997.14248086	32.79881154	36	30.40	0.0001
TREAT	6	1025.7746468	32.18069880	36	31.88	0.0001
TREAT	7	899.98399457	31.04376591	36	28.99	0.0001

DISCUSSION

First consider the SAS/GLM outputs. It appears that no matter which formulation of the model is used, identical results are obtained for all formulations. The following items were compared:

Treatment 7 mean = 900.8
Treatment 1 - 7 effect = 128.8105
Standard error of a difference for the preceding = 45.500149
Residual mean square = 7,352
Type III treatment mean square = 19,954

All formulations gave identical results as shown in Table 1.

The story for the SAS/MIXED procedure is completely different. Some of the differences observed are listed below:

Model	Mean Treat 7	Standard error	Variance component row	column	residual
Row-col.	899.1	69.58	6,871	21,141	7,347
Centr. OP	896.1	29.99			7,071
Noncen. P	not obtainable				

Using the "type = toep(1)" command for row for column regressions, the following was obtained:

Centr. OP	899.1	31.02	6,871	21,141	7,347
Noncen. P	not obtainable				
Cci & crj	880.0	31.51	6,871	21,141	7,347
Pci & prj	900.0	31.04	7,140	21,138	7,355
Xci & xrj	898.8	31.10	13,146	35,462	7,387

The standard error for the row-column formulation listed with the lsmeans is wrong. This can be verified by observing the standard error of a differences of two effects obtained by adding "/solution" after the word "treat" in the model statement. This value can also be compared to the one obtained using the SAS/GLM procedure (See Table 1.). It is amazing that the Helmert formulation with the cci and crj obtains the same variance components yet obtains different means and standard errors.

It is worthy of noting that the standard errors with values like 31.10 are standard errors of a mean but that the standard errors listed for effects are standard errors of a difference of two means or two effects. It should also be noted that when the centered orthogonal polynomials were pooled by rows and by columns, the same means and variance components were obtained as for the row-column formulation.

Murari Singh, ICARDA, Aleppo, Syria, was kind enough to run models 1, 2, and 3 on GENSTAT. From Table 1, we see that SAS/MIXED and GENSTAT/VCOMP obtain the same results for model 1 bur not for model 2. The latter package obtained results for model 3 whereas SAS/MIXED did not. The GENSTAT means were higher than SAS for model 2. For model 3, GENSTAT lsmeans were much lower than for the other models. Hence, different formulations of a model and different packages give different results resulting in a mixed model software muddle.

The above code was run on a three row by four column design with four treatments. The results are given in Table 2 for a number of characteristics. Different formulations of the same model gave different results. One item of note is that SAS/MIXED did produce results for non-centered polynomials for this smaller example. The code for this example and the results in Table 2 is given in the Appendix.

Table 1. Summary for eight row by seven column example.

	Treatment mean									Variance component		
Model	1	2	3	4	5	6	7	SE	SED	Res	Row	Col
GLM												
1,2,3	1030	1053	1051	1032	997	1023	901		44-46	7352		
MIXED												
1sas	1031	1054	1051	1030	996	1025	899	70-70	44-46	7347	6871	21141
1gen	1031	1054	1051	1030	996	1025	898		44-49	7346	6875	21140
2sas	1030	1052	1060	1038	990	1022	896	30-32	43-44	7071		
2gen	1043	1065	1073	1050	1003	1036	910		44-49	7352		
3sas										11137		
3gen	873	884	885	850	831	840	729		45-52	8008		
MIXED - pooled into one source												
1	1032	1055	1051	1029	996	1026	898	68-69	44-46	7340	13492	
2	1032	1055	1051	1029	996	1026	898	31-33	44-46	7340	13492	
3										33835	-.0000	
MIXED - pooled by rows and by columns												
2	1031	1054	1051	1030	996	1025	899	31-33	44-46	7347	6871	21141
3										27662	.0000	.0007
GLM - restraints on parameters; H = Helmert contrasts, P = orthogonal Polynomial; L = effect minus last effect												
1H	1030	1053	1051	1032	997	1023	901		44-46	7352		
1L	1030	1053	1051	1032	997	1023	901		44-46	7352		
1P	1030	1053	1051	1032	997	1023	901		44-46	7352		
MIXED - pooled; restraints are H = Helmert contrasts, P = orthogonal Polynomial, L = last effect equal zero.												
2	1031	1054	1051	1030	996	1025	899	31-33	44-46	7347	6871	21141
1H	1012	1035	1032	1011	977	1006	880	32-34	44-46	7347	6871	21141
1L	1033	1056	1051	1028	995	1025	899	31-34	44-46	7387	13146	35462
1P	1032	1055	1052	1031	997	1026	900	31-33	44-46	7355	7140	21138

SE = standard error listed with lsmeans.

SED = standard error of a difference listed with treatment estimates (effects).

Table 2. Summary for three row by four column design with four treatments.

	Treatment mean						Variance component		
Model	1	2	3	4	SE	SED	Res	row	column
GLM									
1	23.7	26.9	29.7	31.2			38.27		
2	23.7	26.9	29.7	31.2			38.27		
3	23.7	26.9	29.7	31.2			38.27		
MIXED									
1	24.6	27.4	28.7	30.7	6.26	5.21	38.27	66.69	14.32
2	24.2	26.7	29.1	31.4	2.96	4.1-4.2	25.52		
3	17.6	20.0	19.8	22.1	6.57	5.67	64.82		
MIXED - pooled into one source of variation									
1	24.2	27.2	29.2	30.9	5.94	5.18	36.81	38.10	
2	24.2	27.2	29.2	30.9	3.62	5.18	36.81	38.10	
3	17.6	20.0	19.8	22.1	6.31	8.91	119.3	-.000	
MIXED - pooled for rows and for columns									
1	24.6	27.4	28.7	30.7	6.26	5.21	38.27	66.69	14.32
2P	24.6	27.4	28.7	30.7	3.65	5.21	38.27	66.60	14.32
3P	17.5	19.9	19.7	22.0	5.84	6.59	65.19	3.14	0.00

APPENDIX

The following is the code for a three-row by four column data set with four treatments:

```
/*The SAS code for obtaining standard textbook analysis and PRTA are:
 height */
data colrow;  input Yield row col Trt;
pc1=col; pc2=col**2; pc3=col**3; pr1=row; pr2=row**2;
datalines;
19.0 1  1  1
20.3 1  2  2
17.7 1  3  3
36.3 1  4  4
24.7 2  1  2
19.0 2  2  3
22.3 2  3  4
23.3 2  4  1
46.7 3  1  3
31.7 3  2  4
34.3 3  3  1
39.0 3  4  2
run;


/*--code for non-centered polynomials--*/


 proc print;
 run;
```

```
/*--code to construct orthogonal polynomials--*/
Proc iml;
/*--7 columns and up to 6th degree polynomials--*/
opn4=orpol(1:4,3);
opn4[,1]=(1:4)`;
 op4=opn4;       print op4;
 create opn4 from opn4[colname={'col' 'c1' 'c2' 'c3'}];
 append from opn4;
 close opn4;      run;
opn3=orpol(1:3,2);
 opn3[,1]=(1:3)`;
 op3=opn3;     print op3;
 create opn3 from opn3[colname={'row' 'r1' 'r2'}] ;
 append from opn3;
 close opn3;       run;
/*--merge in polynomial coefficients--*/
 data rcbig;
 set colrow;
 idx = _n_;
 proc sort data = rcbig;
  by col ;
 data rcbig ;
  merge rcbig opn4;
  by col ;
 proc sort data = rcbig;
  by row ;
 data rcbig ;
  merge rcbig opn3;
  by row ;
 proc sort data = rcbig ;
  by idx ;
run;

/*--ANOVA for row-column design--*/
Proc Glm data = rcbig;
 Class row  col  Trt ;
 Model Yield = row  col  Trt ; lsmeans Trt;
 run ;

 Proc Glm data = rcbig ;
  Class Trt row  col  ;
  Model Yield = c1 c2 c3  r1 r2 Trt ;
  lsmeans Trt ;
run;

Proc Glm data = work.colrow ;
  Class Trt row col;
  Model Yield = pc1 pc2 pc3  pr1 pr2 Trt;
  lsmeans Trt; run;
Proc Mixed data = rcbig ;
  Class row  col  Trt ;
  Model Yield = Trt/solution ;
  Random row  col;
  Lsmeans Trt ;
run;
Proc mixed data = rcbig;
```

```
  Class Trt row col ;
  Model Yield = Trt/solution; random c1 c2 c3  r1 r2;
             lsmeans Trt; run;
Proc mixed data = work.colrow ;
  Class Trt row col ;
  Model Yield = Trt/solution;
  Random pc1 pc2 pc3  pr1 pr2 ;
  Lsmeans Trt;
run;
Proc mixed data = rcbig;
  Class Trt row col ;
  Model Yield = Trt/solution;
  Random row col/type = toep(1);
  Lsmeans Trt; run;
Proc mixed data = rcbig; Class Trt row col; Model Yield=Trt/solution;
random  c1 c2 c3    r1 r2  /type=toep(1);
 lsmeans Trt; run;

 Proc mixed data = work.colrow;
  class Trt row col;
  Model Yield = Trt/solution;
  Random pc1 pc2 pc3  pr1 pr2 /type=toep(1);
  lsmeans Trt; run;
Proc mixed data = rcbig;
  Class Trt row col;
  Model Yield = Trt/solution;
  Random c1 c2 c3/type = toep(1) ;
  Random r1 r2 /type = toep(1) ;
  Lsmeans Trt ; run ;
Proc mixed data = work.colrow ;
  Class Trt row col ;
  Model Yield = Trt/solution ;
  Random pc1 pc2 pc3 /type = toep(1) s cl ;
  Random pr1 pr2 /type = toep(1) ;
  Lsmeans Trt ; run ; quit;
```